

Expected Value

Example: Flip a coin : if heads, you pay me \$1.00;
if tails, I pay you \$3.00.

How much do you expect to make on average in the long run?

Play 20 times: Should get about ___ heads & ___ tails.

You pay me \$1.00 ___ times : ___

I pay you \$3.00 ___ times : ___

You come out _____

↳ ___ per game

_____ of your winnings

Review & generalize:

Outcome	Value	Hypothetical Frequency	Hypothetical Gains/Losses	Hyp. Net Gain	Expected Net Gain Per Game
heads				}	 per game
tails					

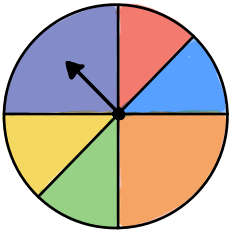
$$\frac{(\quad) \times (\quad) + (\quad) \times (\quad)}{\quad} = (\quad) \times (\quad) + (\quad) \times (\quad)$$

↑ ↑ ↑ ↑

Expected Value

- 1) Assign _____ to outcomes
- 2) Determine _____ of outcomes
- 3) _____ values & probabilities, then _____ to get expected value.

Example: Simple carnival game: prize wheel



Orange: win prize worth \$1.00
Purple: win prize worth \$2.00
Yellow: " " " \$3.00
Green: " " " \$4.00
Blue: " " " \$5.00
Red: " " " \$10.00

Assume probability is proportional to size of colored slice:

$P(\text{Orange}) = \underline{\hspace{1cm}}$ $P(\text{Yellow}) = \underline{\hspace{1cm}}$ $P(\text{Blue}) = \underline{\hspace{1cm}}$
 $P(\text{Purple}) = \underline{\hspace{1cm}}$ $P(\text{Green}) = \underline{\hspace{1cm}}$ $P(\text{Red}) = \underline{\hspace{1cm}}$

What is the expected value of your prize?

$(\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) + (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) + (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) + (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) + (\underline{\hspace{1cm}})(\underline{\hspace{1cm}}) + (\underline{\hspace{1cm}})(\underline{\hspace{1cm}})$
 $= \underline{\hspace{2cm}}$

Person who owns game might charge \$5.00 to play. They know they would, on average, in the long run, make _____ per game.

Broader Applications

Weighing costs & benefits, analyzing risk

- business, insurance, day to day life, big decisions, ..., Wordle...

Ex Current job pays \$60,000 a year. Start my own business?

- might lose \$100,000; 5% chance
- might break even; 20% chance
- likely come out ahead \$40,000; 60% chance
- might make it big \$80,000; 15% chance

Expected value :

$$(\quad) \times (\quad) + (\quad) \times (\quad) + (\quad) \times (\quad) + (\quad) \times (\quad) + (\quad) \times (\quad)$$

$$= \quad \quad (\quad \text{ than current income})$$

How much do you want to start your own business?

What other factors do you need to consider?

Note: Can give numerical values that are not monetary.

Ex: Wordle

S	T	A	R	E

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

If I try my guess words one at a time :

<u>Outcome</u>	<u>Value (Score)</u>	<u>Probability</u>
stage	_____	_____
stake	_____	_____
stale	_____	_____
state	_____	_____
stave	_____	_____
other	_____	_____

Expected Score :

$$(.195)(2) + (.195)(3) + (.195)(4) + (.195)(5) + (.195)(6) + (.025)(10)$$

$$= \quad \quad$$

Another strategy: Choose a word with several of the possible new letters

stage

stake

stale

state

stave



If no _____ guess stake.

If not stake, guess state.

If I try glove, then stake, then state:

<u>Outcome</u>	<u>Value (Score)</u>	<u>Probability</u>
stage	3	19.5 %
stake	3	19.5 %
stale	3	19.5 %
state	4	19.5 %
stave	3	19.5 %
other	6	2.5 %

Expected Score:

$$(.195)(3) + (.195)(3) + (.195)(3) + (.195)(4) + (.195)(3) + (.025)(6)$$

= _____

What actually happened?

S	T	A	R	E
G	L	O	V	E
S	T	A	G	E

Wordle in 3.

Application: Analytical Pros & Cons List

<u>Pros</u>	<u>How Good? (Scale of 1-10)</u>	<u>How Likely (0-100%)</u>
1) ...		
2) ...		
3) ...		

<u>Cons</u>	<u>How Bad? (From -1 to -10)</u>	<u>How Likely (0-100%)</u>
1) ...		
2) ...		
3) ...		
4) ...		

- * Multiply values by corresponding probabilities.
- * Add up to get a number that expresses how good/bad the consequences are overall.
- * Always do a "gut check."

Making Decisions for God